met at CERN (dressed as Xmas tree...)
string models
first geometric mirror pair

Duality in the quantum Hall system

C. A. Lütken and G. G. Ross

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We suggest that a unified description of the integer and fractional phases of the quantum Hall system may be possible if the scaling diagram of transport coefficients is invariant under linear fractional (modular) transformations. In this model the hierarchy of states, as well as the observed universality of critical

Experimental probes of emergent symmetries in the quantum Hall system

C.A. Lütken [a,b,*], G.G. Ross [b,c]

[a] *Theory Group, Department of Physics, University of Oslo, Norway*
[b] *CERN, CH-1211, Geneva 23, Switzerland*
[c] *Rudolf Peierls Centre for Theoretical Physics, Department of Physics, University of Oxford, United Kingdom*

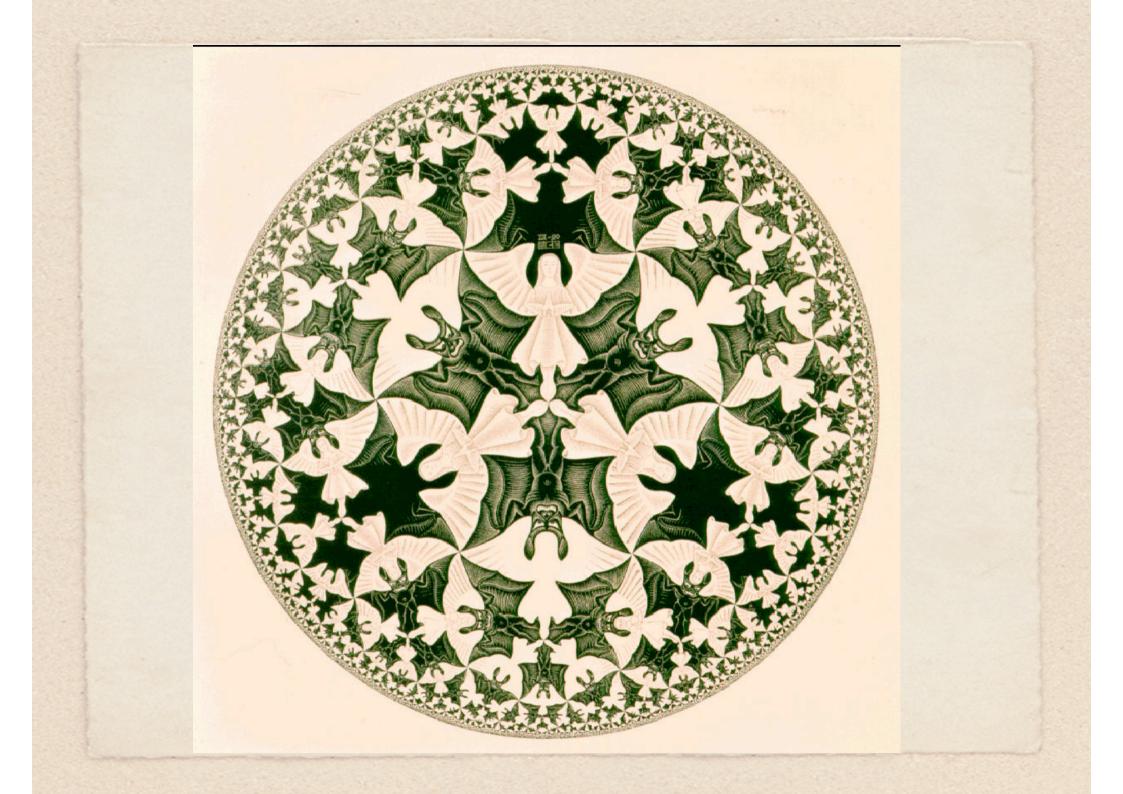# The discovery of

Escher-symmetries

in Nature ?

Escher almost as obsessed with symmetry as we are in physics...
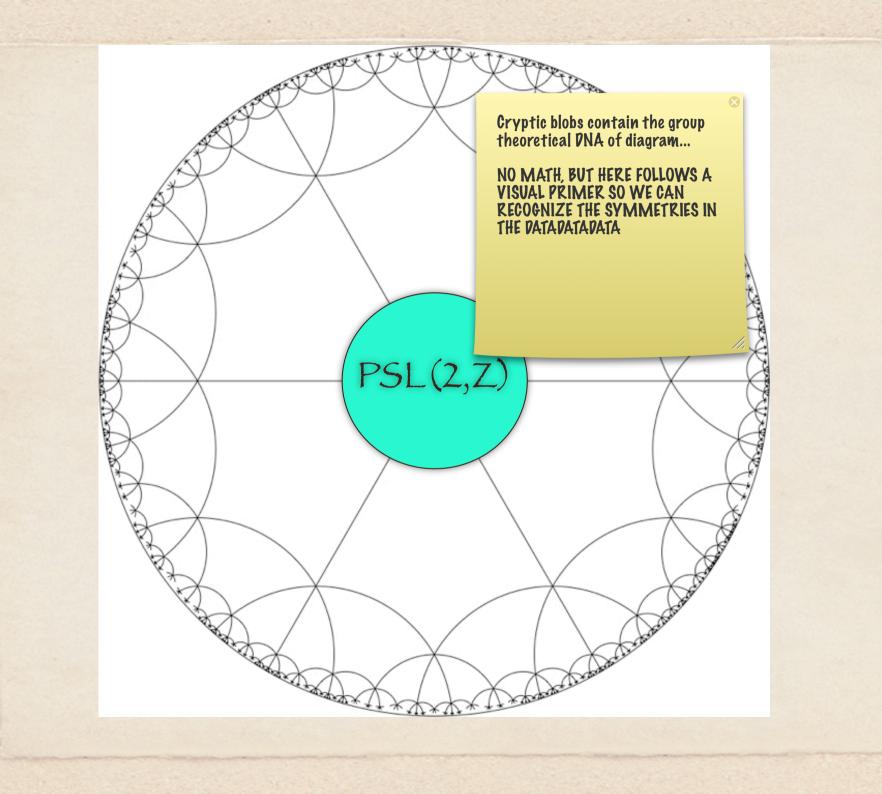
Initially wall-paper symmetries, but after meeting Coxeter in 1954 he introduced hyperbolic (modular) symmetries to the general public.
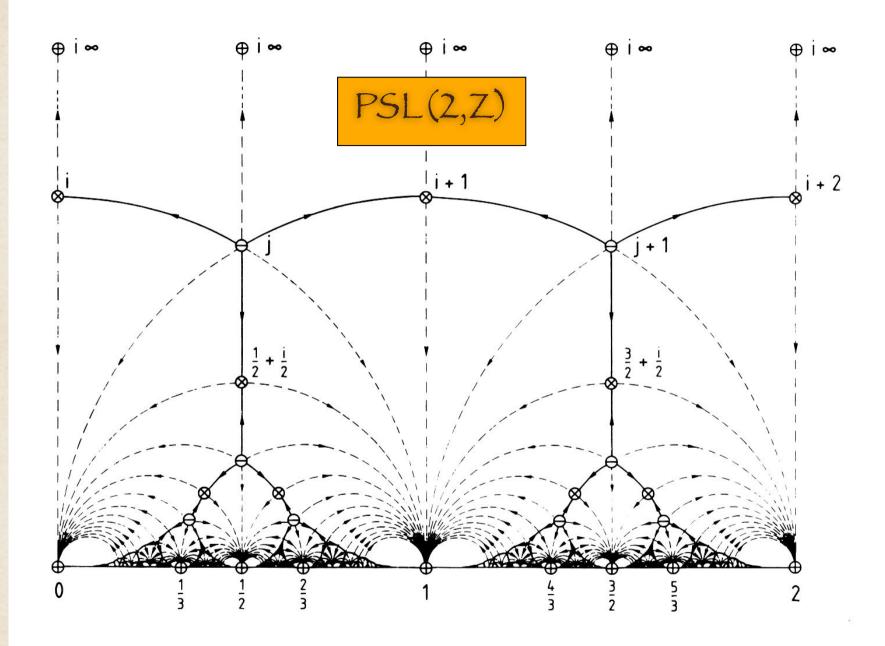
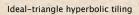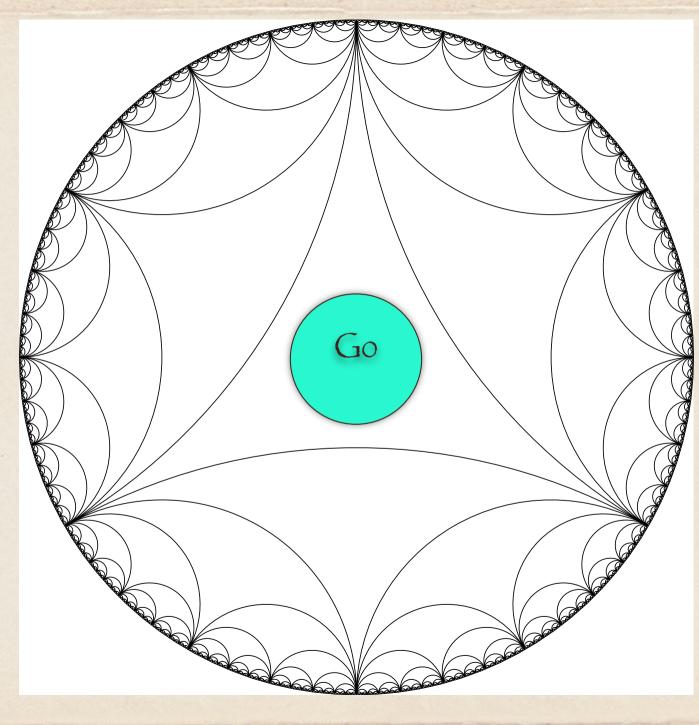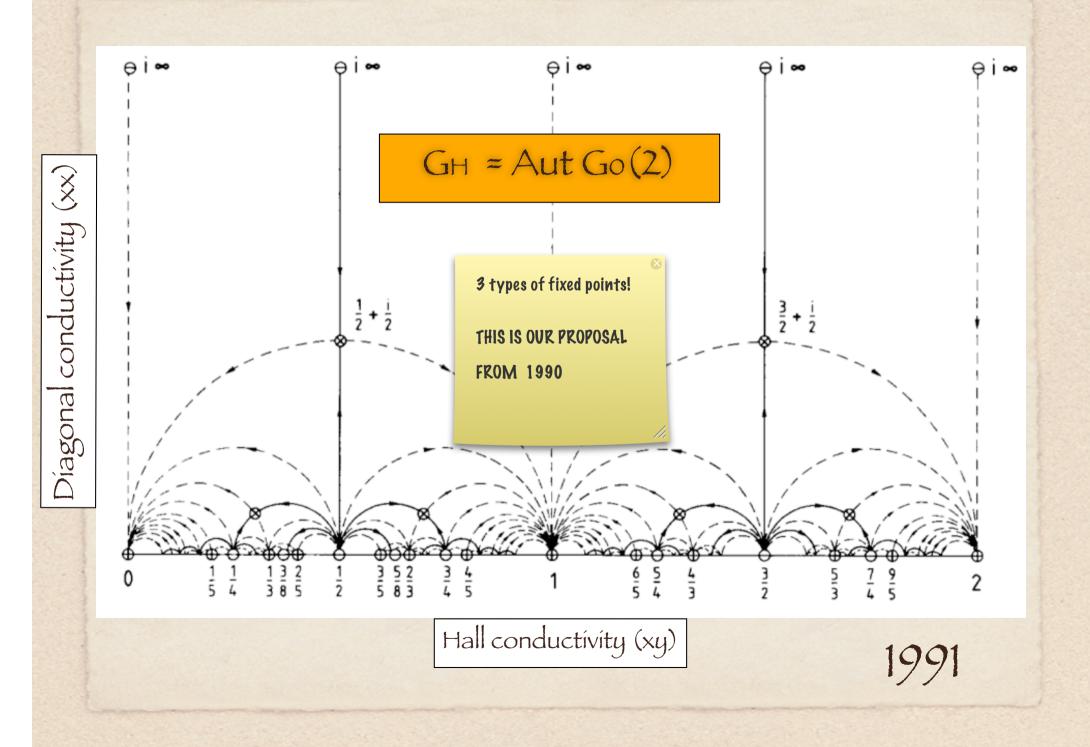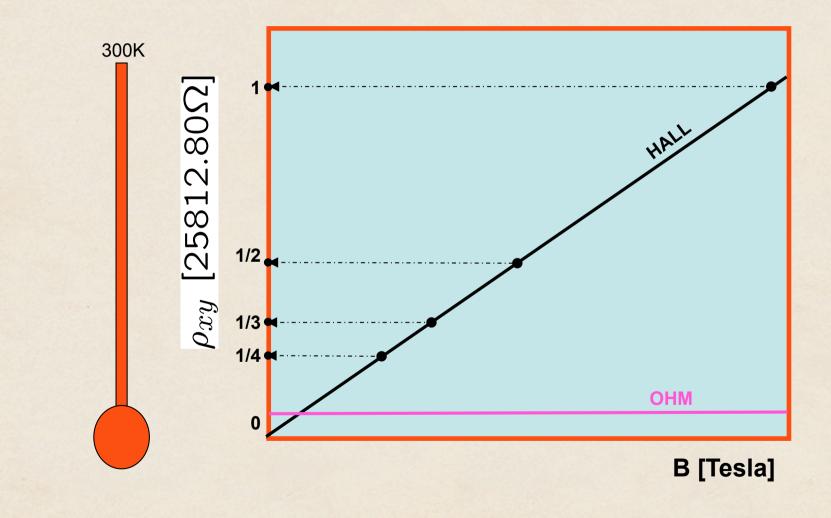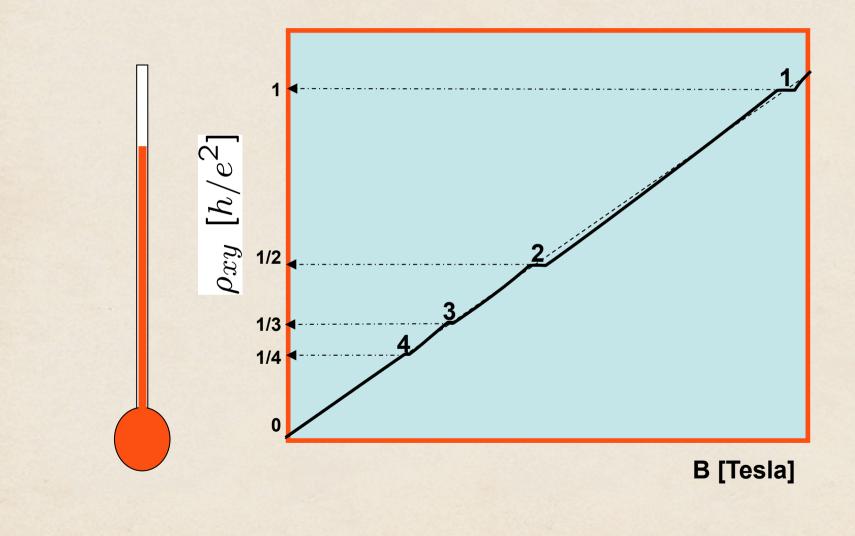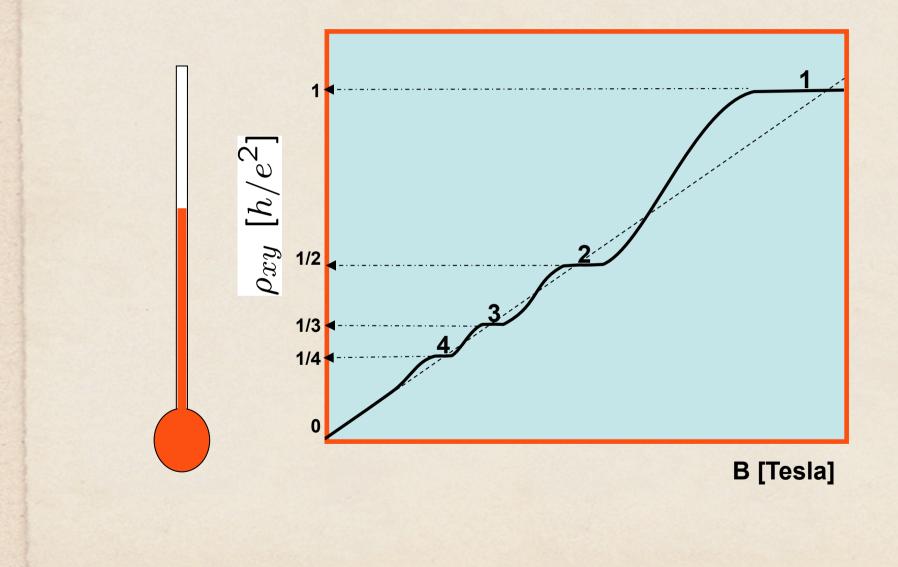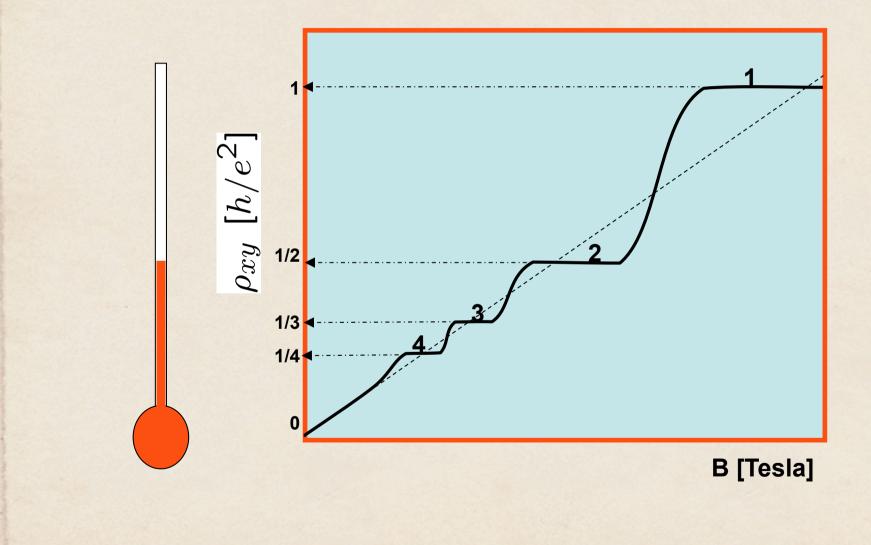Absent from experimental physics.... until now!
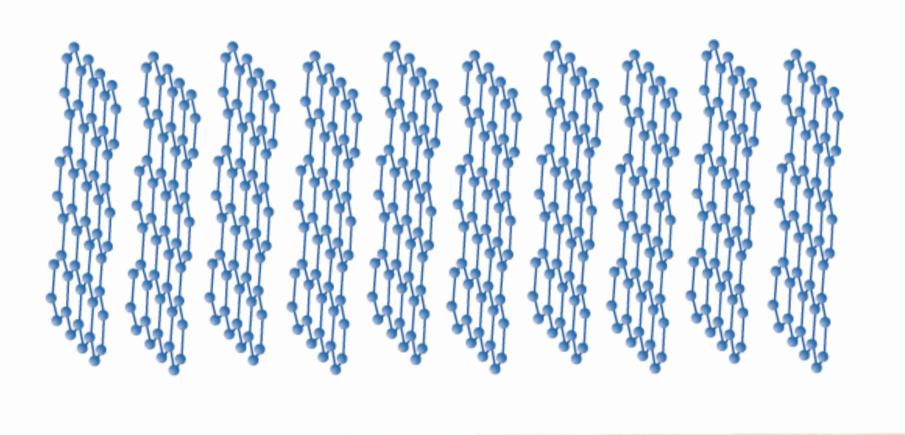
Nature > "Flatland"

The integer QHE

Klaus von Klitzing (1985)

Integer QHE

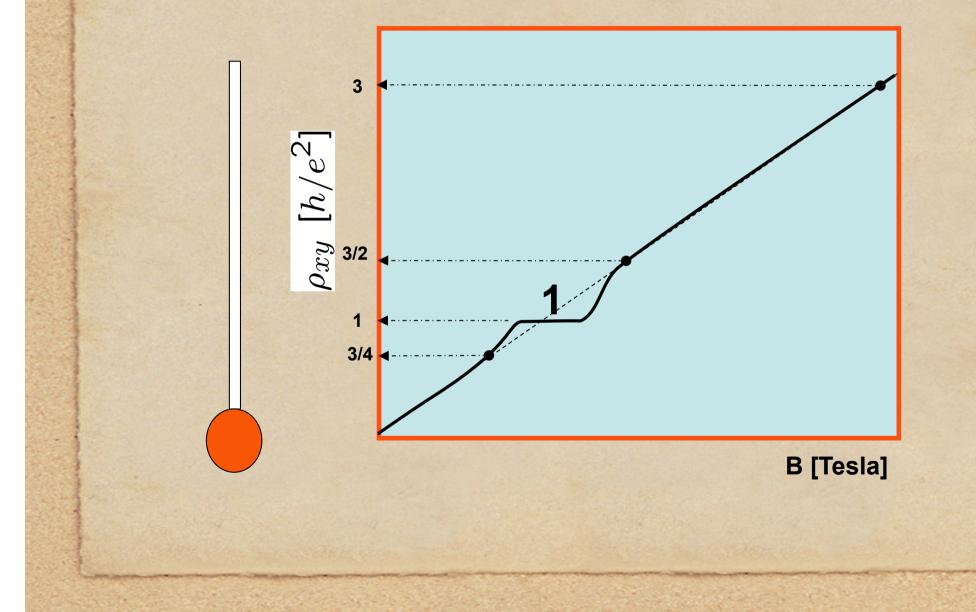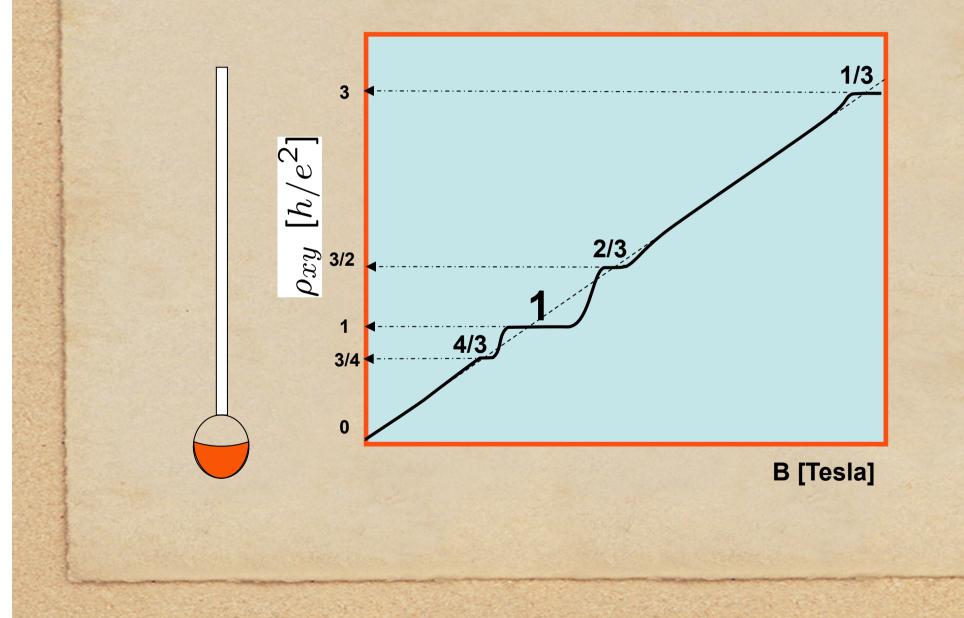ρ_{xy} $[h/e^2]$

1

1/2

1/3

1/4

0

1

2

3

4

B [Tesla]

Integer QHE
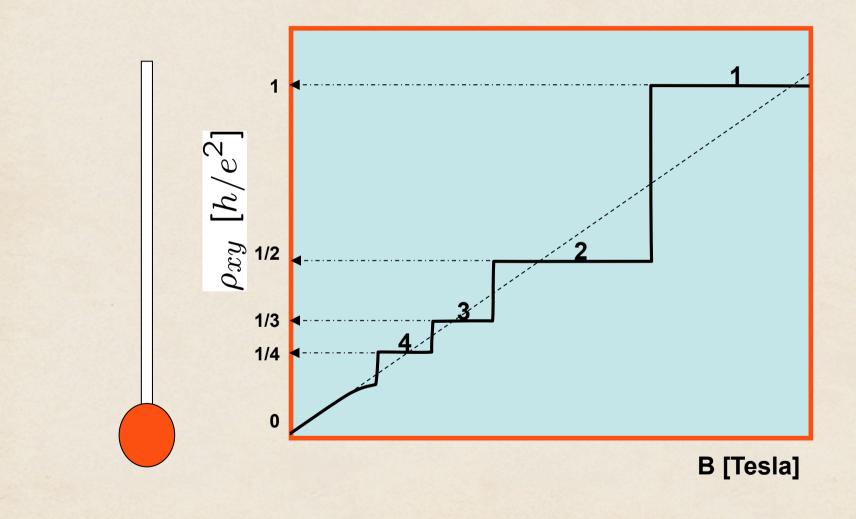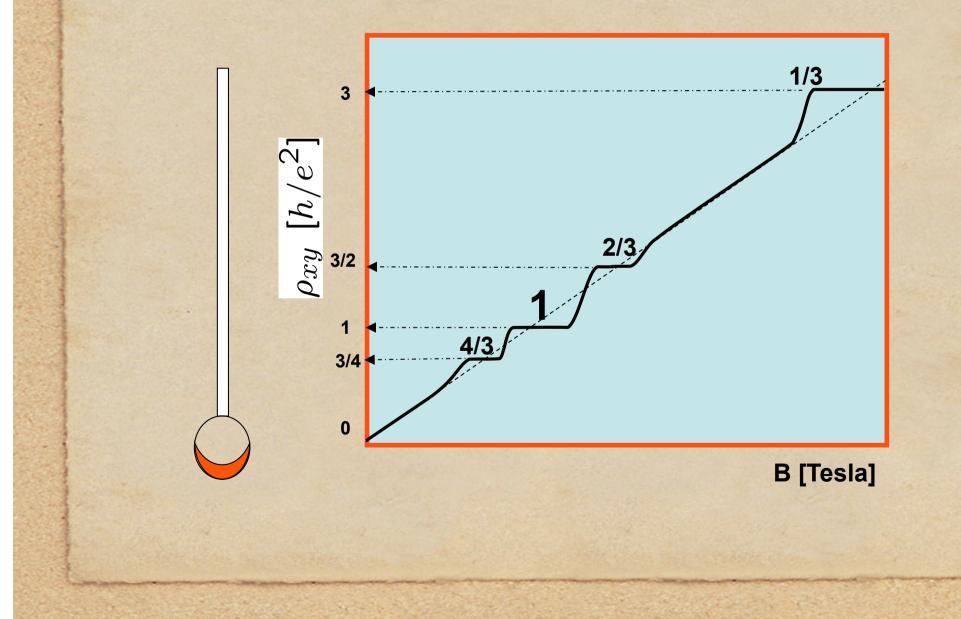
The fractional QHE

Dan Tsui et al. (1998)

$\rho_{xy} \; [h/e^2]$

3

3/2

1

1

3/4

B [Tesla]

Fractional QHE

Fractional QHE

ρ_{xy} $[h/e^2]$

3

3/2

1

3/4

0

1/3

4/3

B [Tesla]

3 TYPES OF RG FIXED POINTS

CONSIDER EACH IN TURN:
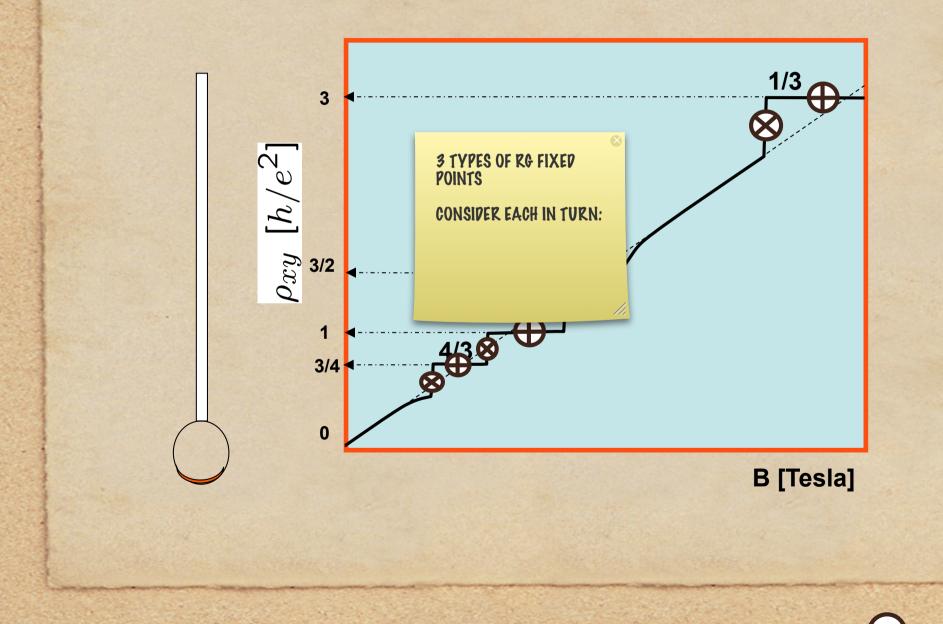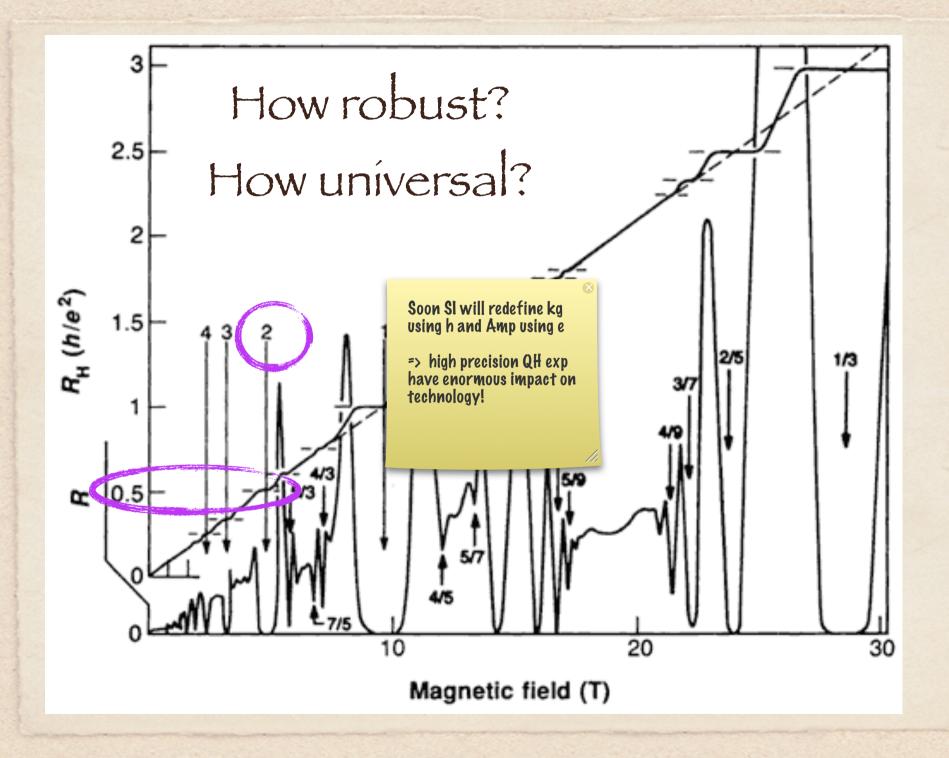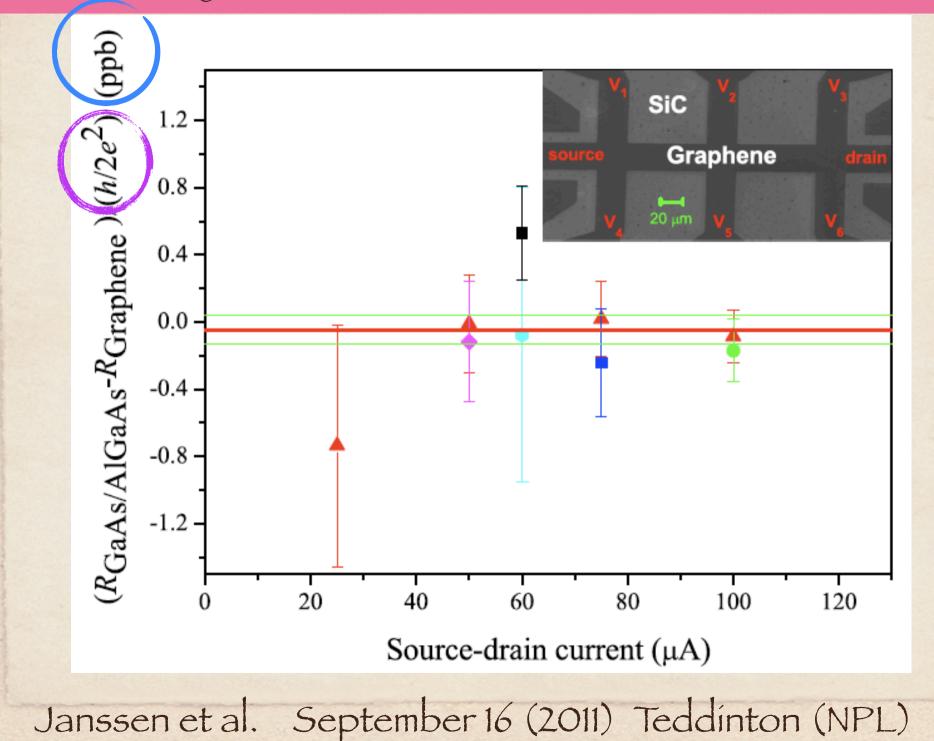
The plateaux:
stable RG fixed points

Bob Laughlin (1998)
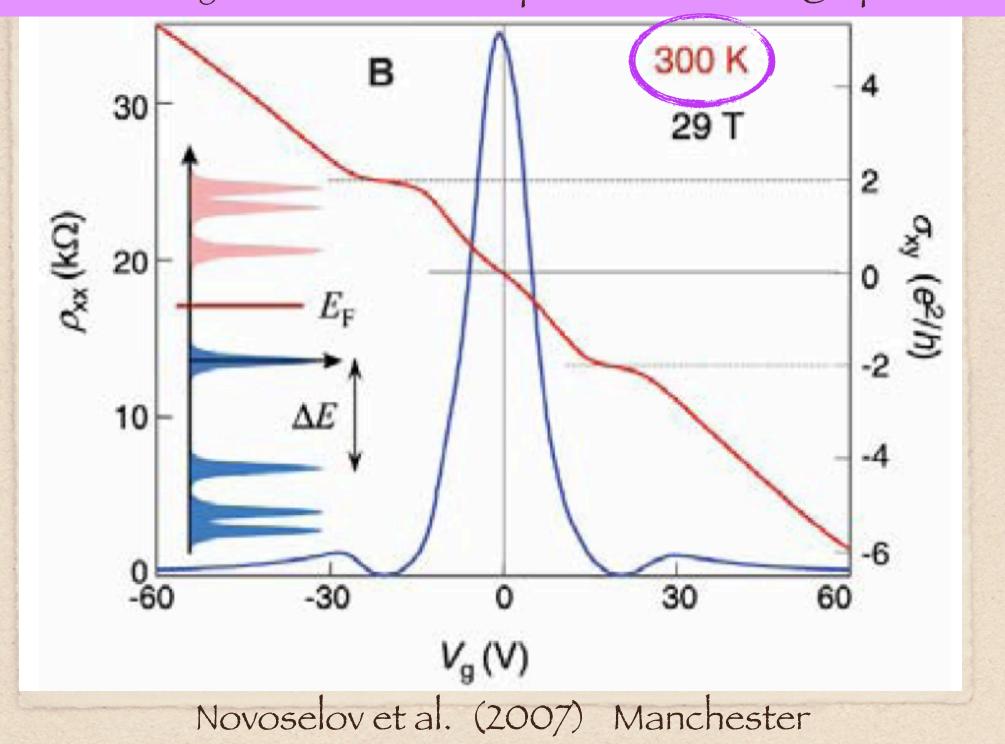
Janssen et al.   September 16 (2011) Teddinton (NPL)

Novoselov et al. (2007) Manchester
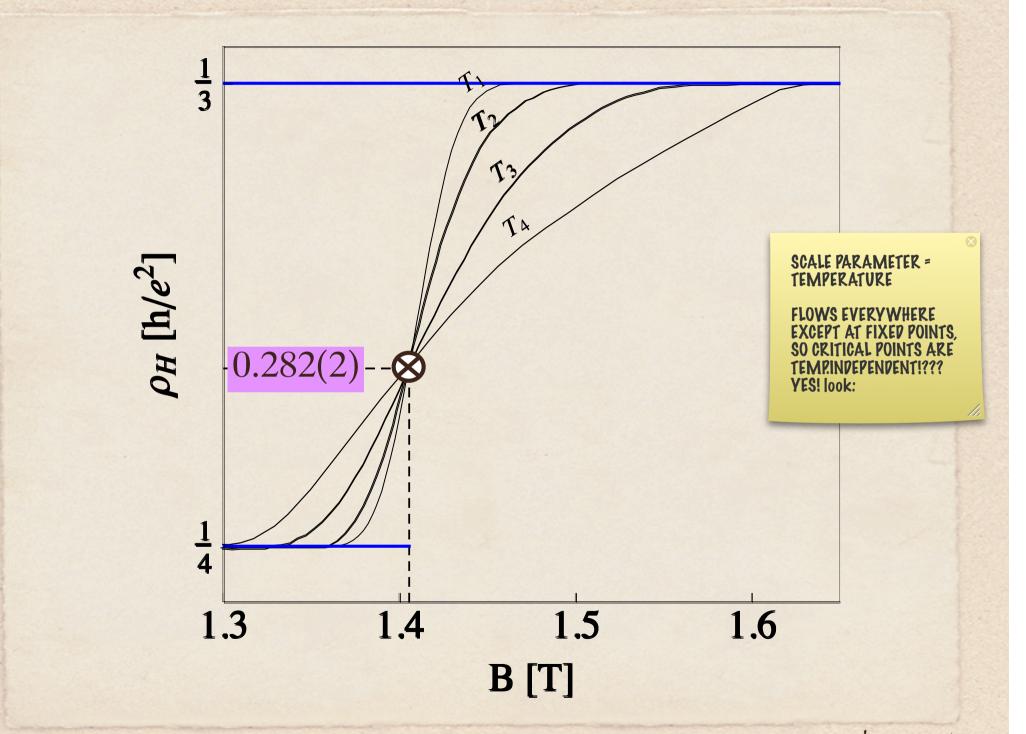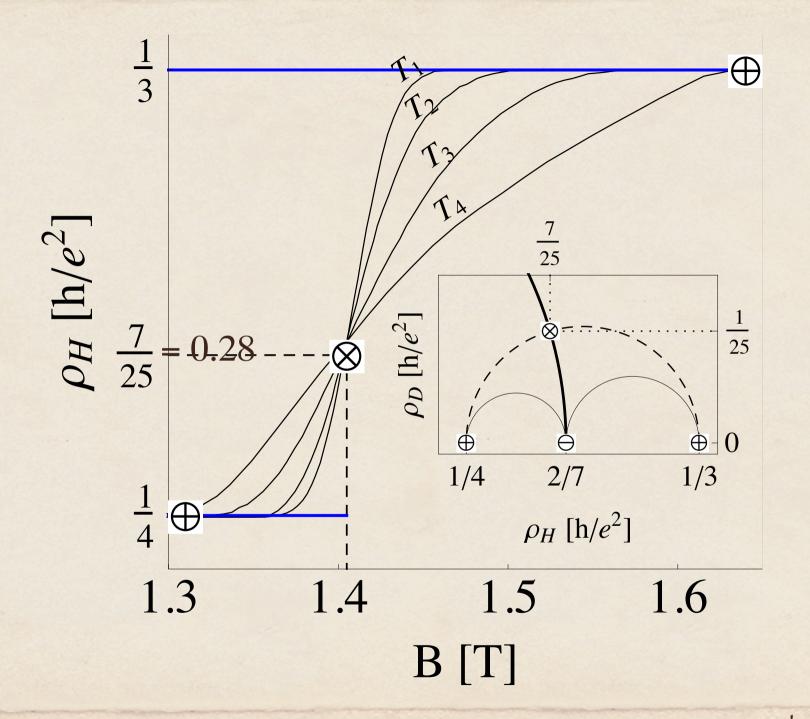
The quantum critical points: semi-stable RG fixed points
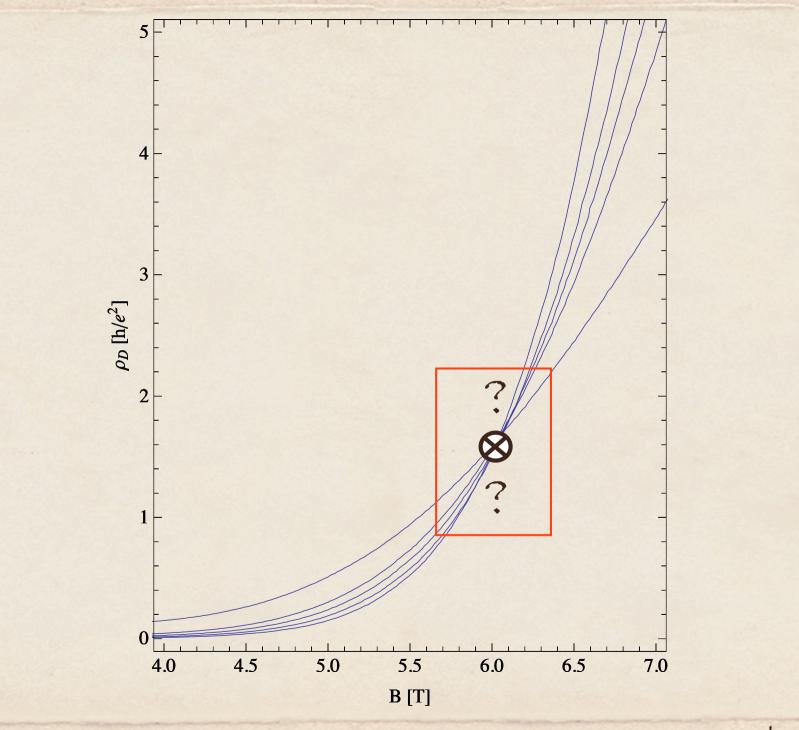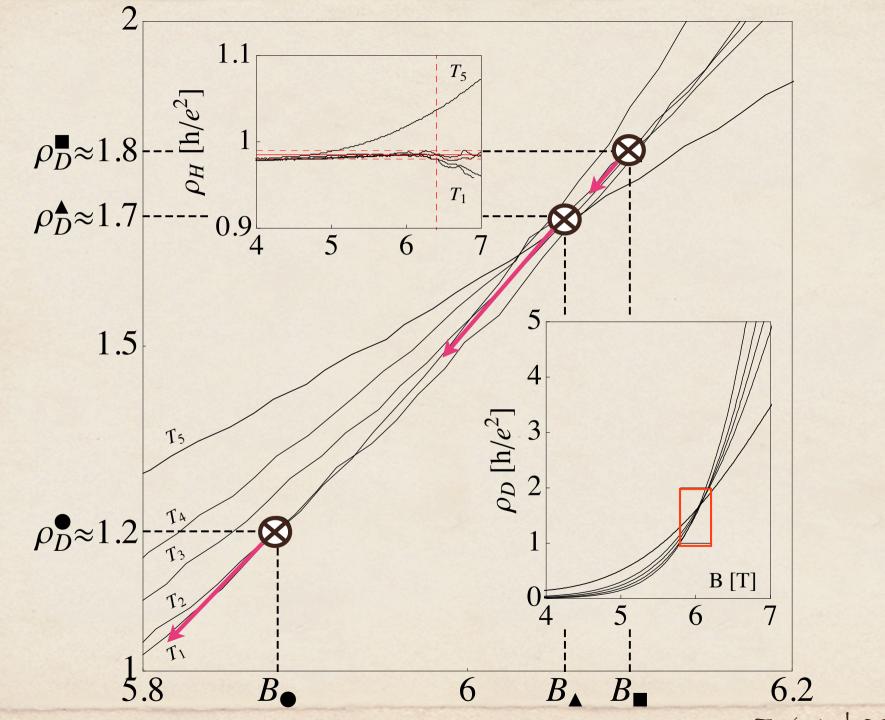
Tsui et al. (2009)
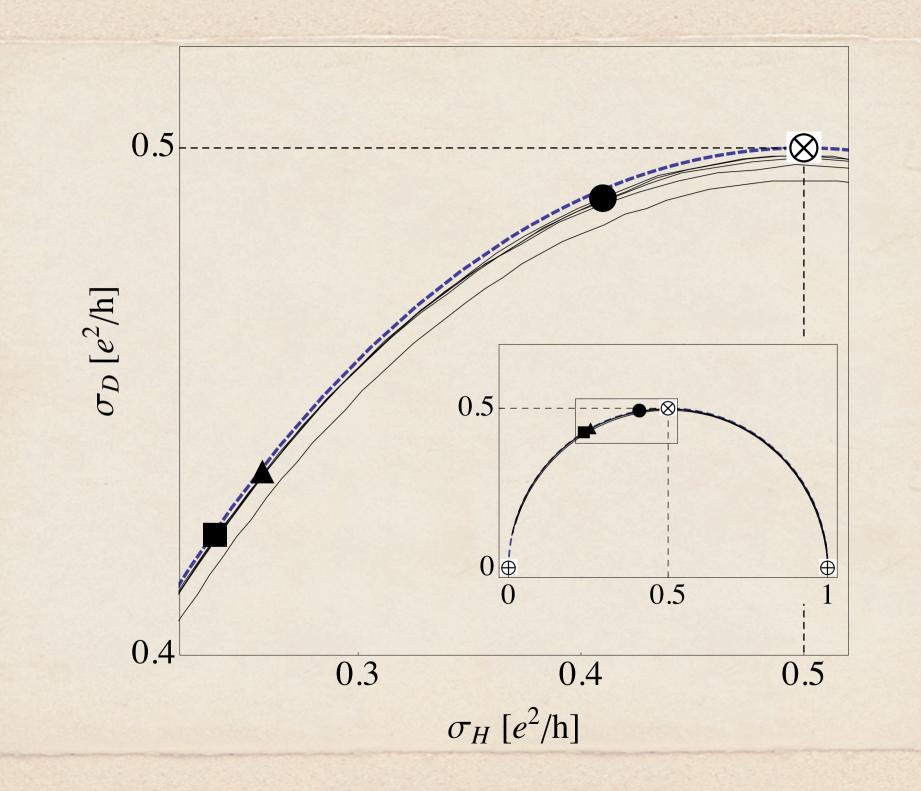
Tsui et al. (2009)
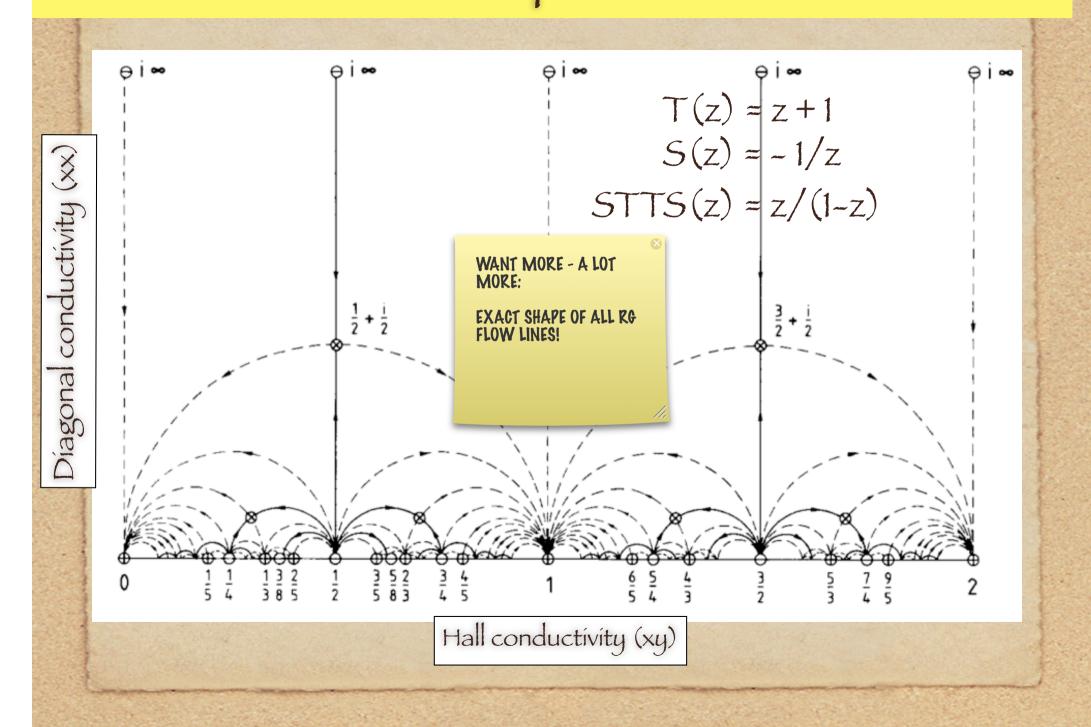
Tsui et al. (2000)

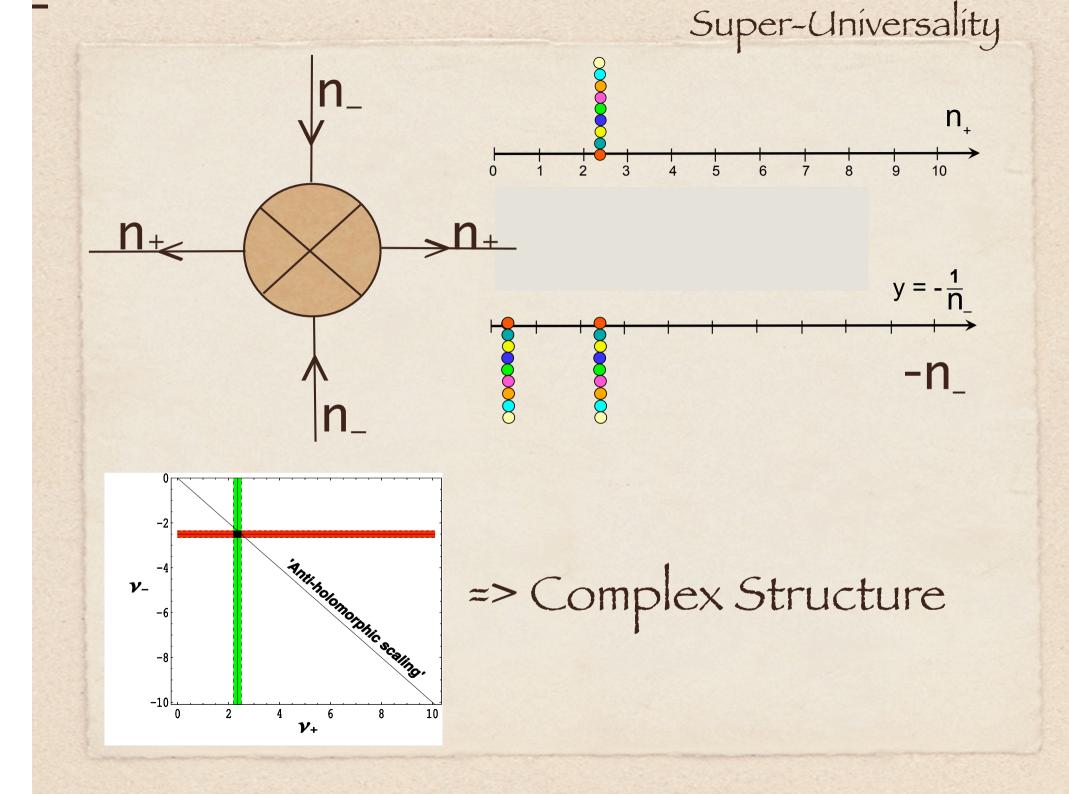Tsui et al. 2000

GH ≈ "DNA" => Phase portait.   RG flow???

Diagonal conductivity (xx)

$$T(z) = z + 1$$
$$S(z) = -1/z$$
$$STTS(z) = z/(1-z)$$

WANT MORE - A LOT MORE:

EXACT SHAPE OF ALL RG FLOW LINES!

Hall conductivity (xy)

$$\beta^1 = \frac{d\sigma_H}{dt}, \quad \beta^2 = \frac{d\sigma_D}{dt}, \quad t = \ln(\Lambda/\Lambda_0).$$

$$\beta^\sigma = \beta^1 + i\beta^2.$$

$$\cancel{\beta^{\sigma'}} = \left(\frac{d\sigma'}{d\sigma}\right)\beta^\sigma = (c\sigma + d)^{-2}\beta^\sigma$$

$$\beta_{\sigma'} = \left(\frac{d\sigma}{d\sigma'}\right)\beta_\sigma = (c\sigma + d)^{+2}\beta_\sigma$$

$$\beta^{\mathrm{phys}} = \beta^\sigma = G^{\sigma\overline{\sigma}}\beta_{\overline{\sigma}}$$

$$\beta_\sigma = -\partial_\sigma\Phi \quad \text{holomorphic???}$$

n_-

n_+

n_+

n_-

n_+

$$y = -\frac{1}{n_-}$$

$-n_-$

'Anti-holomorphic scaling'

ν_-

ν_+

=> Complex Structure

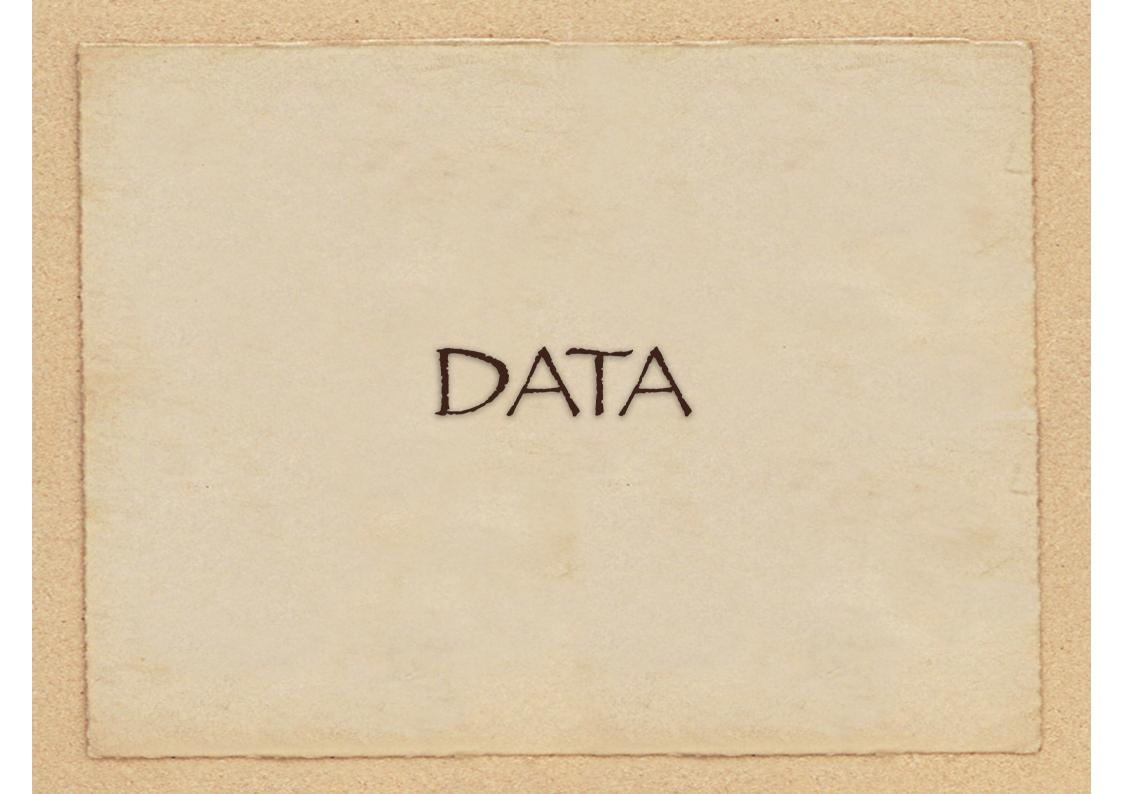$$\Phi_a \propto \ln \lambda (\lambda - 1)^{a-1} + \text{c.c.}, \quad a \in \mathbb{R}$$
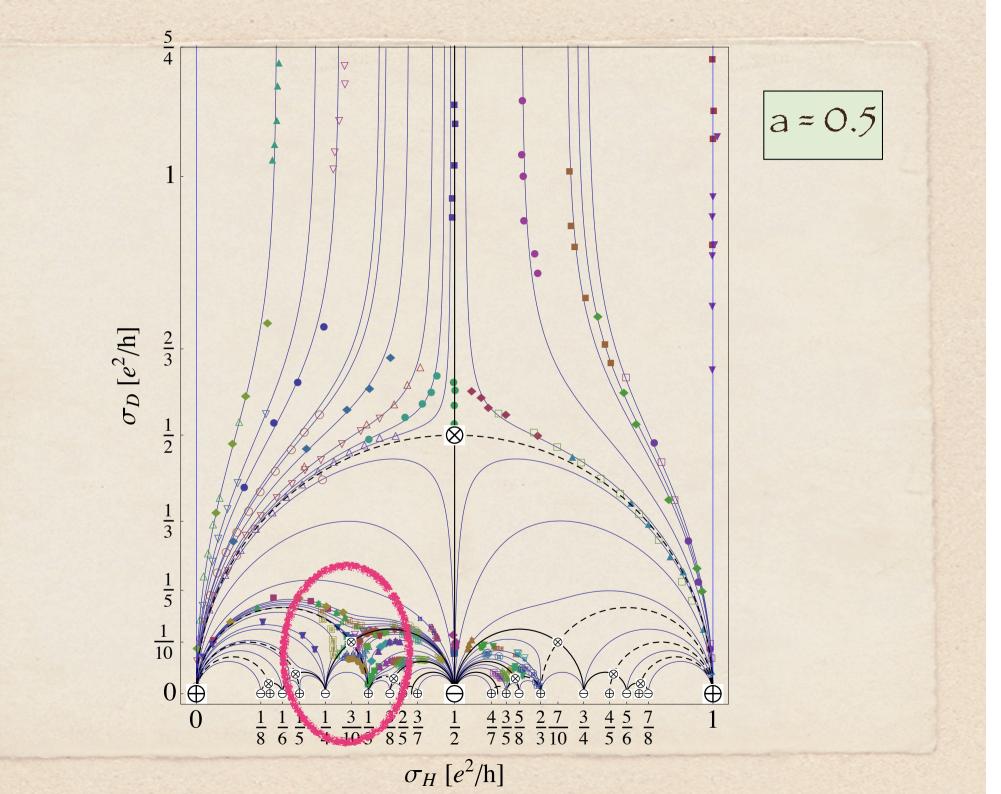
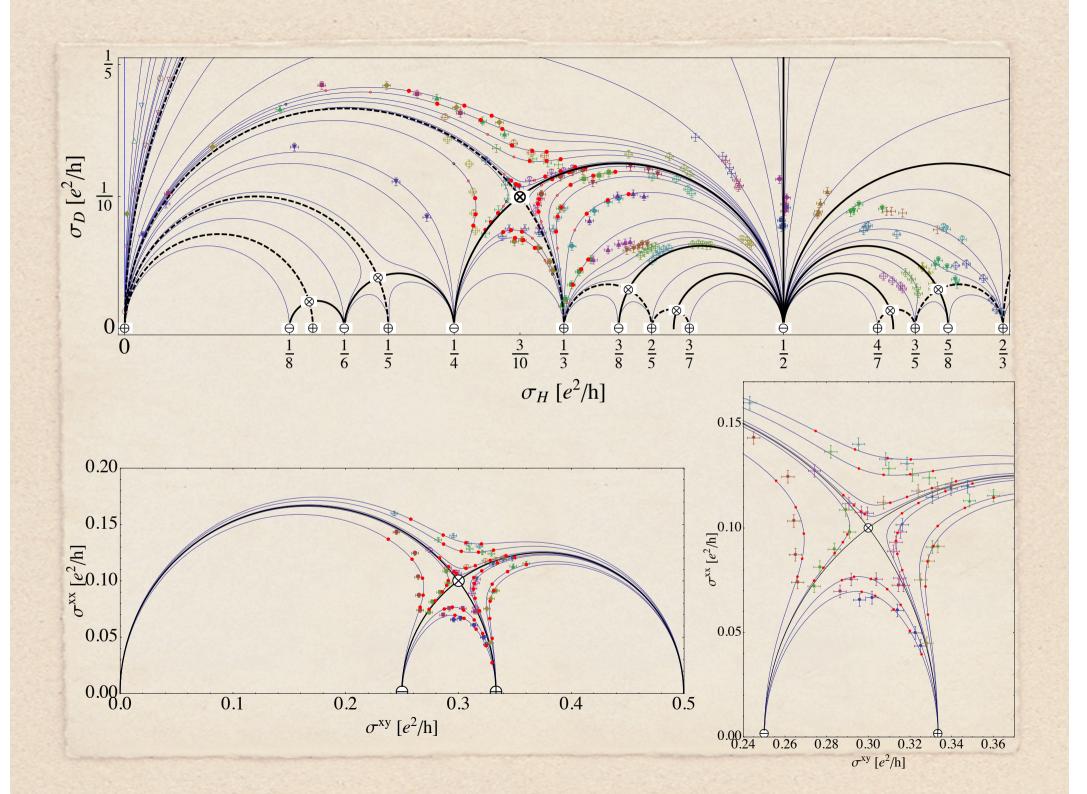$$\lambda(\sigma) = \frac{\theta_2(\sigma)^4}{\theta_3(\sigma)^4}$$

$$\theta_2(\sigma) = \sum_{n=-\infty}^{\infty} q^{\frac{1}{2}(n+1/2)^2}$$

$$q = e^{2\pi i \sigma}$$

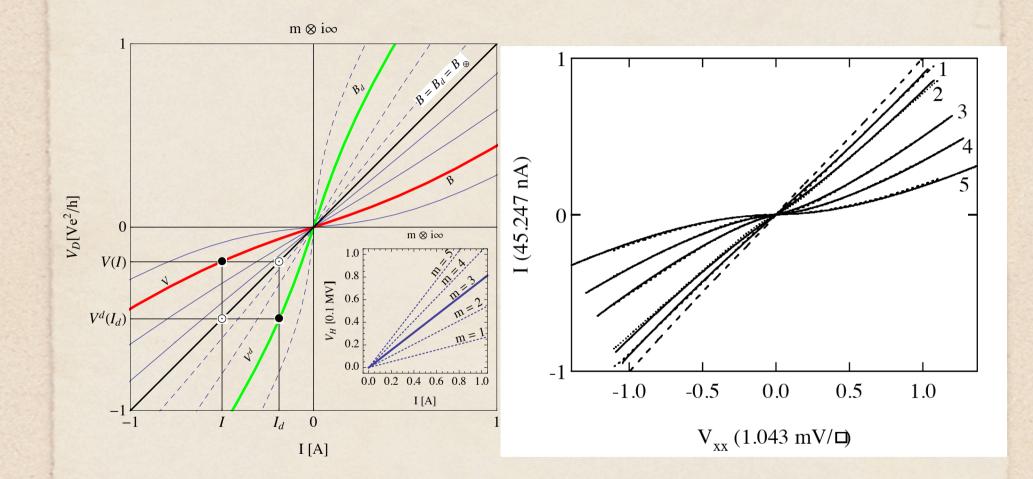$$\theta_3(\sigma) = \sum_{n=-\infty}^{\infty} q^{\frac{1}{2}n^2}$$
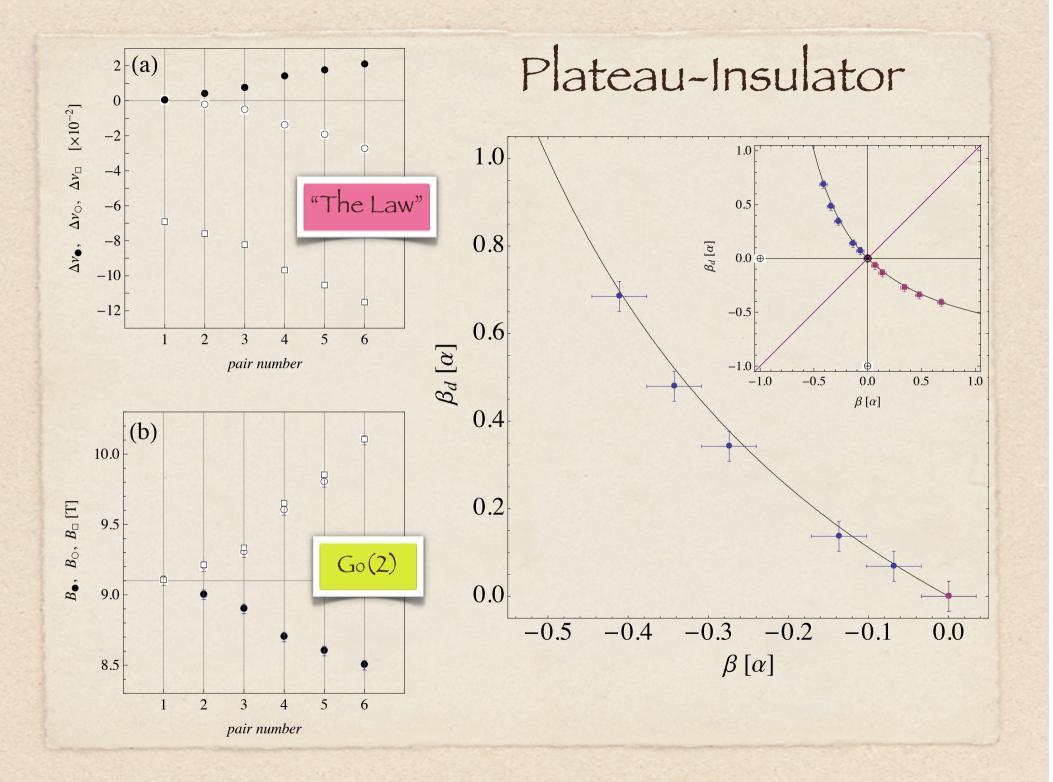
DATA

Duality?

Plateau-Insulator

Tsui et al, Science, 1996

Plateau-Insulator

HYPERBOLIC   TRIANGLE

Roman mosaic in Real Alcazar, Cordoba, Spain